# AP Calculus AB
# Test: Limits and Continuity
# Instructor's Version

**Free-Response Question 1 (no calculator)**
***You may not use your calculator for this question***
(When you finish this question, if your test is proctored, ask your proctor for the next section. If you're taking the unproctored version of the test, continue with the next question.)

**1.** For each of the following find:

I. $\lim_{x \to a-} f(x)$

II. $\lim_{x \to a+} f(x)$

III. $\lim_{x \to a} f(x)$

**A.** $f(x) = \dfrac{|x^2 + 3x + 2|}{x^2 - 4}$ for $a = 2$

Answer: $\lim_{x \to a-} f(x) = -\infty$, as the denominator's values are negative and approaching zero, while the numerator's values approach 3. (1.25 points)

$\lim_{x \to a+} f(x) = \infty$, as the denominator's values are positive and approaching zero, while the numerator's values approach 3. (1.25 points)

$\lim_{x \to a} f(x)$ = DNE because the one-sided limits are unequal. (1.25 points)

Note: Award no more than 1.25 points if no justification given.

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**B.** $f(x) = \begin{cases} \sin x, & \text{if } x < \frac{\pi}{6} \\ \tan x, & \text{if } x = \frac{\pi}{6} \\ \cos x, & \text{if } x > \frac{\pi}{6} \end{cases}$ $\qquad a = \frac{\pi}{6}$

Answer: $\lim_{x \to a-} f(x)$ = ½ , as the value of sine is ½ at $\frac{\pi}{6}$, but the function does not actually attain this value. (1.25 points)

$\lim_{x \to a+} f(x) = \sqrt{3}/2$ , as the value of cosine is $\sqrt{3}/2$ at $\frac{\pi}{6}$, but the function does not actually attain this value. (1.25 points)

$\lim_{x \to a} f(x)$ = DNE because the one-sided limits are unequal. (1.25 points)

Note: Award no more than 1.25 points if no justification given.

**C.** $f(x) = \begin{cases} \sin\frac{x}{3}, & \text{if } x \le \pi \\ \frac{x\sqrt{3}}{2\pi}, & \text{if } x > \pi \end{cases}$ $\qquad a = \pi$

Answer: $\lim_{x \to a-} f(x) = \frac{\sqrt{3}}{2}$ , as the piece of the graph for values of $x \le \pi$ approaches and reaches this value. (1.25 points)

$\lim_{x \to a+} f(x) = \frac{\sqrt{3}}{2}$, as the piece of the graph for values of $x > \pi$ approaches this value as $x$ nears $\pi$. (1.25 points)

$\lim_{x \to a} f(x) = \frac{\sqrt{3}}{2}$ because the one-sided limits are equal. (1.25 points)

Note: Award no more than 1.25 points if no justification given.

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**D.** $f(x) = \dfrac{x^2 - 36}{\sqrt{x^2 - 12x + 36}} \qquad a = 6$

Answer: $\lim_{x \to a-} f(x)$ = -12, as the numerator's values for the ($x$-6) factor are opposite the denominator's values. (1.25 points)

$\lim_{x \to a+} f(x)$ = 12, as the ($x$-6) factors in the numerator and denominator have the same sign. (1.25 points)

$\lim_{x \to a} f(x)$ = DNE because the one-sided limits are unequal. (1.25 points)

Note: Award no more than 1.25 points if no justification given.

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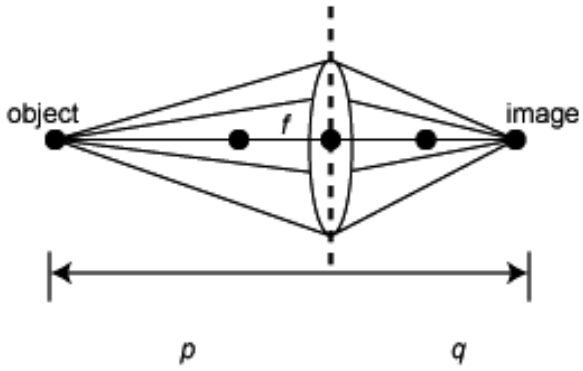
**2.** A convex lens with focal length $f$ centimeters will project the image of an object on a point behind the lens. If an object is placed a distance of $p$ centimeters from the lens, then the distance $q$ centimeters of the image from the lens is related to $p$ and $f$ by the lens equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$




**A.** If the focal length of the convex lens is supposed to be 5 cm. and if the image is formed 7 cm from the lens, find the distance from the lens to the object, $p$. (It's not necessary to simplify your answer.)

Answer: $\frac{1}{5}=\frac{1}{p}+\frac{1}{7} \Rightarrow \frac{1}{5}=\frac{7+p}{7p} \Rightarrow 7p=35+5p \Rightarrow p=\frac{35}{2}cm$

(1 point for correctly plugging in values; 2 points for $p$-value; 1 point for units)

**B.** Find an expression that gives $q$ as a function of $p$, assuming that the focal length is a constant of 5 centimeters.

Answer: $\frac{1}{5}=\frac{1}{p}+\frac{1}{q} \Rightarrow \frac{1}{5}=\frac{q+p}{pq} \Rightarrow pq=5q+5p \Rightarrow q=\frac{5p}{p-5}$

(1 point for rewriting lens equation with $f$ = 5 plugged in; 2 points for work in solving for $q$)

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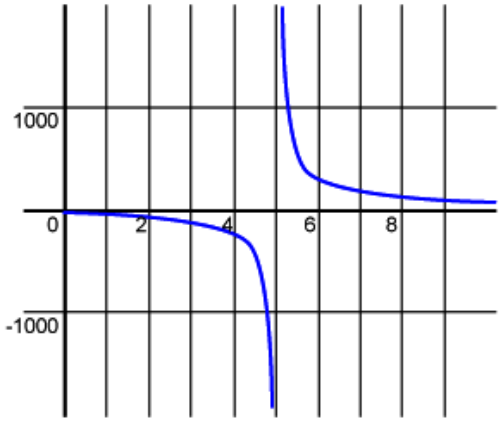
**C.** Sketch a graph of $q$ as a function of $p$ (i.e, $q(p)$), assuming that the focal length is a constant of 5 centimeters. Show any important features of the graph.




Answer: Graph is worth 4 points. Be generous as students do *not* have the use of their calculators. Assign points as follows:

1 point: asymptote $p = 5$
1 point: asymptote $q = 5$
2 points: overall scale/shape

**D.** Find $\lim_{p\to\infty} q(p)$ and $\lim_{p\to 5+} q(p)$. What do these limits represent physically? What must happen to the distance of the image and the object?

Answer: $\lim_{p\to\infty} q(p) = 5$ means that as the distance of the object from the lens grows larger and larger, the location of the image formed approaches the lens since the focal length is fixed.

$\lim_{p\to 5+} q(p) = \infty$ implies that if the object moves toward the focal point on its side of the lens, the image formed moves farther and farther away from the lens until it can no longer be seen.

(1 point for $\lim_{p\to\infty} q(p)$, 1 point for $\lim_{p\to 5+} q(p)$, and 2 points for justification/explanations)

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